Icelandic High School Math Contest 2025–2026
Final

NOTE: Turn in separate pages for each problem. Only write on one side. Put your name on all pages. This is important for grading. No calculators. Logically support all answers in written form.

Dæmi 1
You start with a (finite) list L of numbers. You replace this list L with a new list that counts how often each numbere appeared in L. For example if a number appeared thrice in L the new list contains a three. If $L = [7, 1, 1, 1, 4, 5, 4]$ then the new list is $[1, 1, 2, 3]$. Then we replace the new list with a newer one in the same way, repeatedly. Prove that the list eventually converges to containing just a single copy of the number 1.

Dæmi 2
Gísli really likes the hot tub he has out in the countryside. There are three faucets for it, one red, one green and one blue. The flow rate and temperature of each faucet is fixed.

1. If the red and green faucets are turned on it takes 70 minutes to fill the tub with 41 °C water.
2. If the red and blue faucets are turned on it takes 42 minutes to fill the tub with 35 °C water.
3. If the green and blue faucets are turned on it takes 35 minutes to fill the tub with 39 °C water.

If we turned on all the faucets at once, what would the resulting water temperature in the hot tub be once it is full? (***Note**: If one liter of $20\,^\circ C$ water is mixed with two liters of $29\,^\circ C$ water you get three liters of $26\,^\circ C$ water.*)

Dæmi 3
A right-angled triangle with positive integer side lengths has circumference 1000. What could its side lengths be?

Dæmi 4
Magnea, Mía and Friðfinnur sit in a circle. A positive integer has been written on each of their foreheads, so they each see the other numbers but not their own. They are told that the sum of two of those numbers equals the third. Magnea is asked what her number is and she answers she does not know. Mía is asked the same, same answer. Friðfinnur is asked, same answer. Finally Magnea is asked again what her number is and answers 65. What are the numbers on their foreheads? They all know the numbers are positive integers. They are all incredibly smart and know that the others are incredibly smart.

Dæmi 5
Let Γ be a circle and P be some point outside that circle. Let the two tangents from P to the circle Γ touch Γ at the points A, B. Show that the center of the inscribed circle of ABP is on Γ.

Dæmi 6
A hacker is hiding in one of 2^N servers, each of which is specified by N bits. For example if $N = 5$ one server is specified with 01101. To catch him, K policemen connect to K servers. Then the hacker and police alternate taking moves. They may choose to stay in place or move to a server with one bit different, for example one can move from 01101 to 11101. What does K need to be at a minimum to ensure a policeman can be at the same server as the hacker after a finite amount of time? You may assume everyone employs the best strategy possible. Everyone knows where everyone is. All police move at once, but each policeman can choose individually whether they move or not.